# RAMAKRISHNA MISSION VIDYAMANDIRA

**(A Residential Autonomous College)**

Belur Math, Howrah

B.A./B.Sc. 1st Semester (July – December 2010)

**Mid-Semester Examination, September 2010**

Date: 06.09.2010 **Physics (Honours)** Full Marks 50

Time: 11 am – 1 pm

**Answer all questions.**
**Use two scripts. One for Gr-A and other for Gr-B**

## Group-A (Mathematical Method )

1. a) Find the series solution of the equation, $y'' - 2xy' - 4y = 0$ such that $(y)_{x=o} = 0$ and $(y')_{x=o} = 1$. 4

   b) Assuming the solution of the equation $(1-x^2)y'' - 2x\,y' + n(n+1)\,y = 0$ in the form $y(x) = \sum_{r=0}^{\infty} a_r\, x^{k+r}$, show that k (k-1) = 0. 3

   c) Given that $(\sqrt{1-2xt+t^2}\,)^{-1} = \sum_{n=0}^{\infty} t^n P_n(x)$, show that $P_n(1) = 1$ for all values of n. 3

2. a) State the Fourier theorem for expansion of a periodic function. 2

   b) Can $y = (\cos x)^{-1}$ be expanded in a Fourier series ? Explain. 1+2

   c) Write down the complex form of Fourier Series. 2

3. a) Consider the alternating series: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots\ldots$

   Show that it is not absolutely convergent, but converges. 3

   b) Show that the Pfaffian [ xy cos(xy) + sin(xy) ] dx + [$x^2$ cos( xy) ] dy is an exact differential . Find suitable function whose differential is the given Pfaffian. 4

4. a) Determine whether the following vectors are linearly independent or not :

   i) $\vec{A} = [1,1,0],\ \vec{B} = [0,1,1],\ \vec{C} = [1,0,1]$

   ii) $\vec{A} = [1,-6,2],\ \vec{B} = [0,2,7],\ \vec{C} = [-2,12,-4]$ 2

   b) If $\vec{a_1}, \vec{a_2}, \vec{a_3}$ are nonzero, non-coplanar vectors, then show that any vector $\vec{A}$ can be represented as $\vec{a_1}(\vec{A}\cdot\vec{b_1}) + \vec{a_2}(\vec{A}\cdot\vec{b_2}) + \vec{a_3}(\vec{A}\cdot\vec{b_3})$, where $(\vec{a_1}, \vec{a_2}, \vec{a_3})$ and ( $\vec{b_1}, \vec{b_2}, \vec{b_3}$) are reciprocal sets of vectors. 2

   c) Find the directional derivative of $\phi(x,y,z) = x^2 + y^2 + z^2$ in the direction from P (1,1,0) to Q (2,1,1). Find also its maximum value and direction at (1,1,0) . 3

d) i) If $f(x,y,z)$ is a scalar field and $\vec{A}(x,y,z)$ is a vector field , prove that

$\text{div}(f\vec{A}) = f\,\text{div}\vec{A} + \text{grad}(f)\,\vec{A}$ 3

**OR**

ii) For an arbitrary constant vector $\vec{a}$ , and the position vector $\vec{r}$ , prove

$\text{grad}(\vec{a}.\vec{r}) = \vec{a}$ 3

## Group-B

5. a) What is optical path ? State Fermat's principle. 1+1

b) Show that to produce a perfect parallel beam of light a parabolic mirror should be used. 2

c) For refraction of light from a medium of refractive index $n_1$ at a curved of radius of curvature R to a medium of refractive index $n_2$, Show by Fermat's principle that $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ where u and v are object and image distances respectively. 4

6. a) What do you mean by spherical aberration ? Mention one method for its remedy. 2+1

b) Show that a single lens of finite focal length can not be made achromatic. 2

c) Deduce the achromatic condition for two lenses in contact. 3

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